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Some properties of intersection points of Euler line and orthotriangle

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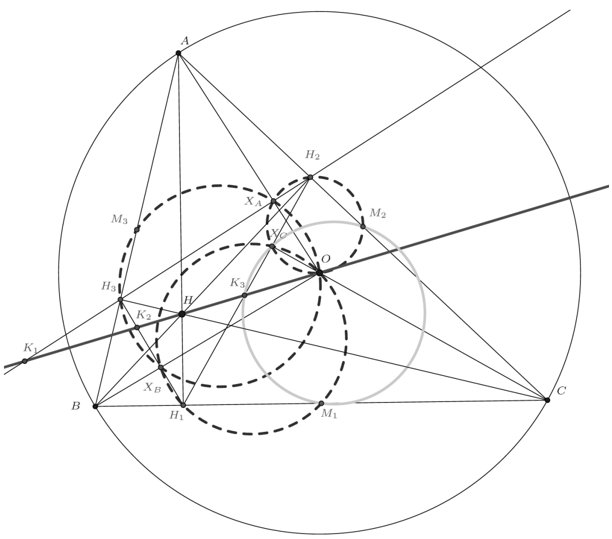
Geometry is an important part of the problems set at the International Mathematical Olympiad. For the last few years geometry problems have often been placed on the 3rd or the 6th position which are the places for the toughest problems and this tendency seems to be here to stay. This proves that for their complexity and beauty geometry problems are in the range of current interests of the school mathematical education.

In the report we present an investigation which was initiated by the problem inspired by IMO2013 3 and IMOSL2012 G6 construction. While solving this problem, we discovered several facts about intersection points of the Euler line and the sides of orthotriangle. Further investigation of these points resulted in facts which we find intersting on their own and want to share them.

What is the main problem?



Let ABC be an acute triangle. Its altitudes AH_1, BH_2, CH_3 intersect at point H. Denote midpoints of the sides AB, BC, CA by M_3, M_1, M_2 and the circumcenter of ABC by O. Let X_A be foot of perpendicular from A to H_2H_3. Define points X_B, X_C analogously.
The key lemma of IMOSL2012 G6 and IMO2013 3 states the following: the circumcircles of triangles $M_1X_BX_C, M_2X_AX_C, M_3X_AX_B$ intersect at point O. One might wonder, whether a similar statement holds for triangles $M_1M_3X_B, M_1M_2X_C, M_2M_3X_A$. Suprisingly, this is also true. While proving this fact we discover that the intersection point of Euler line and the line H_1H_2 K_3 belongs to the circumcircle of triangle $M_1M_2X_C$ which is a striking fact on its own. Then we study other interesting features of these points and their relations to the known objects.